Mathematical Methods for Computer Science

Consider the $N$-point Discrete Fourier Transform (DFT) of the sequence $f[n]$ for $n = 0, 1, \ldots, N - 1$ given by

$$F[k] = \sum_{n=0}^{N-1} f[n] e^{-2\pi i nk/N}$$

with the inverse DFT given by

$$f[n] = \frac{1}{N} \sum_{k=0}^{N-1} F[k] e^{2\pi i nk/N}.$$

(a) Show that $F[k]$ has period $N$, that is $F[k + N] = F[k]$. \hfill [2 marks]

(b) Derive the shift property in the $n$-domain, namely, that $f[n - m]$ has $N$-point DFT given by $e^{-2\pi i mk/N} F[k]$. \hfill [4 marks]

(c) Define the $N$-point cyclic convolution, $(f * g)[n]$, of two sequences $f[n]$ and $g[n]$ by

$$(f * g)[n] = \sum_{m=0}^{N-1} f[m] g[n - m]$$

and show that $(f * g)[n]$ has $N$-point DFT given by $F[k] G[k]$ where $G[k]$ is the $N$-point DFT of the sequence $g[n]$. \hfill [6 marks]


(i) direct calculation; \hfill [4 marks]


You may assume that the binary operation $*$ distributes over pointwise addition of sequences.