Data Structures and Algorithms

(a) Define the minimum spanning tree (MST) of a graph and justify, with counterexamples where appropriate, why the search for it makes sense only on connected, weighted and undirected graphs. [2 marks]

(b) Define these MST expressions: safe edge, cut, respecting a set of edges. [3 marks]

(c) Describe an efficient MST-finding algorithm, write some clear pseudocode for it and prove its correctness. [9 marks]

(d) Say whether each of the following two statements is true or false, justifying each answer with a proof or a counterexample.

(i) Graph $G$ has a unique MST $\Rightarrow$ For every cut of $G$, the lightest edge that crosses it is unique.

(ii) Graph $G$ has a unique MST $\Leftarrow$ For every cut of $G$, the lightest edge that crosses it is unique. [6 marks]