

2006 Paper 2 Question 4

Discrete Mathematics I

Consider two natural numbers $a, b \in \mathbb{N}$.

(a) Define the highest common factor (a, b) of a and b . [2 marks]

(b) Show that the set of linear combinations of a and b is equal to the set of multiples of their highest common factor:

$$\{as + bt \mid s, t \in \mathbb{Z}\} = \{v(a, b) \mid v \in \mathbb{Z}\} \quad [8 \text{ marks}]$$

Suppose that a and b are co-prime, so $(a, b) = 1$. Define a *natural linear combination* of $a, b \in \mathbb{N}$ to be $as + bt$ with $s, t \in \mathbb{N}_0$.

(c) Show that $ab - a - b$ cannot be expressed as a natural linear combination of a and b . [2 marks]

(d) Show that any natural number greater than $ab - a - b$ can be expressed as a natural linear combination of a and b . [6 marks]

[Hint: recall the complementary function for a linear Diophantine equation.]

(e) Royal Mail first class stamps cost 32p each and second class stamps cost 23p each. What is the largest postage that cannot be paid exactly with first and second class stamps? [2 marks]