Answer five questions.

Submit the answers in five separate bundles, each with its own cover sheet. On each cover sheet, write the numbers of all attempted questions, and circle the number of the question attached.
1 Data Structures and Algorithms

(a) Explain what the heap data structure is, state its defining properties and explain how to convert between the tree and vector representations of a heap. [2 marks]

(b) Describe an optimally efficient algorithm for transforming any random vector into a heap vector and explain why it works. [4 marks]

(c) Using the tree instead of the vector representation for clarity, apply this algorithm to the binary tree isomorphic to the letter vector “P I S K T Z O P V N”, producing a frame-by-frame trace of the execution. For this answer, please use the sheet for part (c), supplied with the question paper, and use a new tree whenever any nodes change. [5 marks]

(d) Explain how to rearrange the heap after having extracted its top so that what remains is still a heap. Follow this procedure to extract the top three values, one by one, from the heap you built, producing a frame-by-frame trace as above. Please use the sheet for part (d). [5 marks]

(e) Describe a way to insert a new value into an existing heap in time $O(\log n)$ where $n$ is the heap size. [4 marks]

Further copies of the sheets for parts (c) and (d) may be obtained from the Invigilator, if required. Ensure that you attach the sheets to the rest of your answer.
2 Computer Design

(a) Early processors used an accumulator to hold an intermediate result whereas more modern processors use a register file or a stack.

(i) Why use a stack or register file rather than an accumulator? [4 marks]

(ii) What are the advantages and disadvantages of using a register file rather than a stack? [6 marks]

(b) In 1946, Burks, Goldstein and von Neumann made the observation “Ideally one would desire an indefinitely large memory capacity such that any particular word would be immediately available. We are forced to recognize the possibility of constructing a hierarchy of memories, each of which has greater capacity than the preceding but which is less quickly accessible.”

(i) Why is this memory design issue even more pressing today? [5 marks]

(ii) What architectural techniques are used today to approximate a large memory which is nearly instantaneously accessible? [5 marks]

3 Digital Communication I

(a) Describe and contrast the processes of (i) forward error correction and (ii) error detection with retransmission. [5 marks]

(b) What properties should be considered when deciding which should be used to control errors? [10 marks]

(c) Are there circumstances when both should be used? Justify your answer. [5 marks]
4 Distributed Systems

Members of an open process group manage distributed replicas of data values stored in persistent memory. To allow the system to operate in the presence of transient failures of some replica managers, a quorum assembly scheme is used. Replica managers are assumed to be non-malicious and fail-stop.

To update a managed data item, the operations provided by the managing process include:

\[
\text{lock(item)} \\
\text{update(item, value, timestamp)} \\
\text{read(item, timestamp)} \\
\text{unlock(item)}
\]

(a) Suppose the data item is an initially empty list of values and the update operation appends a value. Illustrate the quorum assembly scheme for five replicas, showing a number of update and read operations. [8 marks]

(b) How is a total order of updates achieved by quorum assembly in the presence of concurrent update requests by clients to the open group? Discuss how any problems that might arise can be solved. [4 marks]

(c) When can \text{unlock(item)} be executed safely by the initiating replica manager? Describe any additional protocol that is needed. [5 marks]

(d) Suppose that the process group is managing non-overlapping partitions of a distributed database instead of replicas. Can quorum assembly play any part in making the related updates required for distributed transactions? Justify your answer. [3 marks]
5 Computer Graphics and Image Processing

(a) Standard colour printing uses the four ink colours: cyan, magenta, yellow and black.

(i) Explain why this is so. [3 marks]

(ii) What benefits are there in using more than these four ink colours? [3 marks]

(b) Describe an algorithm for converting a greyscale image to a bilevel (black and white) image while retaining as good a quality as possible. Your algorithm should be for the situation where each greyscale pixel maps to a single bilevel pixel. [6 marks]

(c) Describe operations on images which achieve the following effects:

(i) lighten an image which is too dark; [2 marks]

(ii) remove salt and pepper noise (“shot noise”) from an image; [2 marks]

(iii) locate 45° edges in an image; [2 marks]

(iv) convert a colour image (in RGB format) to a greyscale image while preserving the perceived luminance. [2 marks]
6 Compiler Construction

(a) Describe a difference and a similarity between the notions of overloading and polymorphism. [2 marks]

(b) Define the notion of type safety in a programming language. [2 marks]

(c) Describe the linking phase and the difference between static and dynamic linking. [2 marks]

(d) Suppose that a programming language allows nested functions. How might a stack-based implementation of \( f \) access the value associated with \( a \)?

\[
\begin{align*}
\text{int } \text{g}(\text{int } a) \\
\{ \\
\text{int } \text{f}(\text{int } x) \{ \\
\quad \text{return } a + x; \\
\} \\
\quad \ldots \\
\quad \text{f}(2); \\
\quad \ldots \\
\}
\end{align*}
\]

[4 marks]

(e) Describe the low-level code that could be generated by compiling the following code fragment for a stack-based target machine.

\[
\begin{align*}
\text{int } \text{f}(\text{int } a, \text{int } b) \\
\{ \\
\quad \text{int } z = a \ast b; \\
\quad \text{return } a + z; \\
\} \\
\quad \ldots \\
\quad \text{f}(1, 2) \ast \text{f}(3, \text{f}(4, 5)); \\
\quad \ldots
\end{align*}
\]

[10 marks]
7 Comparative Programming Languages

(a) An author writes:

Most successful language design efforts share three important characteristics . . .

1. Motivating Application: The language was designed so that a specific kind of program could be written more easily.

2. Abstract Machine: There is a simple and unambiguous program execution model.

3. Theoretical Foundations: Theoretical understanding was the basis for including certain capabilities and omitting others.

Briefly discuss the merits and/or shortcomings of one of the above three statements of your choice, giving examples and/or counterexamples from procedural, applicative, logical, and/or object-oriented programming languages. [6 marks]

(b) For two programming languages of your choice amongst FORTRAN, Algol, Pascal and C, briefly discuss and evaluate their typing disciplines. Further compare the advantages and disadvantages that their designs impose on the programmer. [5 marks]

(c) Consider the following two program fragments.

\[
\begin{align*}
( \text{defvar x 1} ) & \quad \text{val x = 1} ; \\
( \text{defun g(z) (+ x z)} ) & \quad \text{fun g(z) = x + z} ; \\
( \text{defun f(y)} ) & \quad \text{fun f(y)} \\
(+ (g 1) ) & \quad = g(1) + \\
( \ \text{let} ) & \quad \text{let} \\
( ( x (+ y 3)) ) & \quad \text{val x = y + 3} \\
( \text{in} ) & \quad \text{in} \\
( g( + y x) ) & \quad g(y+x) \\
( ) ) ) & \quad \text{end} ; \\
( f 2 ) & \quad f(2) ;
\end{align*}
\]

What are their respective output values when run in their corresponding interpreters? Justify your answer, explaining it in a conceptual manner. [4 marks]

(d) Outline the key features that a language must have to be called object-oriented. Further, briefly discuss to what extent one programming language of your choice amongst Simula, Smalltalk, C++, and Java has them. [5 marks]
8 Databases

Suppose we have the following relational schema

Person(pid:integer, name:string, street:string, postcode:string)
Car(cid:integer, year:integer, model:string)
OwnedBy(pid:integer, cid:integer)
AccidentReport(rid:integer, damage:integer, details:string)
ParticipatedIn(pid:integer, rid:integer, cid:integer)

where the underlined attributes represent the primary keys of the associated relation. The table OwnedBy implements a relationship between persons and cars using foreign keys. The table ParticipatedIn implements a relationship between persons, accident reports, and cars, where tuple (p, r, c) indicates that the person p was the driver of the car c associated with the accident report r.

(a) Write an SQL query to return those pid’s of persons driving in at least one accident, with no duplicates. [2 marks]

(b) Write an SQL query to return all tuples (pid, c), where c is the number of cars owned by person pid (records where c = 0 do not have to be generated). [2 marks]

(c) Write an SQL query to return all tuples (cid, c), where c is the number of persons owning car cid (records where c = 0 do not have to be generated). [2 marks]

(d) Write a (nested) SQL query to return all tuples (pid, rid) where pid was driving in the accident reported in rid, but the car driven by pid is not owned by pid. [4 marks]

(e) Write an SQL query to return all tuples (rid, c), where c is the number of drivers involved in the accident reported in by rid (records where c = 0 do not have to be generated). [2 marks]

(f) Write an SQL query to return all tuples (rid, c), where c is the number of cars involved in the accident reported in by rid (records where c = 0 do not have to be generated). [2 marks]

(g) Do the functional dependencies implied by the schema imply that the results of queries (e) and (f) will always be the same? Explain. [2 marks]

(h) Perhaps there is something wrong with this schema. How would you fix the schema to ensure that results of queries (e) and (f) would always be the same? [4 marks]
9 Numerical Analysis II

(a) With reference to solution of the differential equation $y' = f(x, y)$, explain the conventional notation $x_n$, $y(x_n)$, $y_n$, $f_n$. [3 marks]

(b) Explain the terms local error, global error, and order of a method. [3 marks]

(c) Without deriving any formulae, describe the general technique for deriving multistep formulae. [2 marks]

(d) Milne’s method uses the multistep formulae

$$y_{n+1} = y_{n-3} + \frac{4h}{3}(2f_n - f_{n-1} + 2f_{n-2})$$

$$y_{n+1} = y_{n-1} + \frac{h}{3}(\tilde{f}_{n+1} + 4f_n + f_{n-1})$$

which each have local error $O(h^5)$. What is the meaning of the term $\tilde{f}_{n+1}$? Suggest a suitable starting procedure and explain how the Milne formulae are used. [6 marks]

(e) Let $x_0 = 0.2$, $y(x_0) = 1.67$, $h = 0.2$ and

$$f(x, y) = 1 + \frac{(y - x)(x + 2)}{x + 1}.$$ 

Suppose the following values of $f_n$ have been generated by the starting procedure: 4.6, 5.6, 7.2 for $n = 1, 2, 3$. Calculate the first required value of $\tilde{f}_{n+1}$ to 2 significant digits. [3 marks]

(f) Contrast Milne’s method with your starting procedure, commenting particularly on stability, efficiency and step size considerations. [3 marks]
10 Bioinformatics

(a) Hidden Markov models (HMM) are widely used in Bioinformatics.

(i) In a HMM when would you use the Baum–Welch algorithm, and when the Viterbi algorithm, and why? Give biologically motivated examples. [8 marks]

(ii) Any machine learning model (such as a HMM) for protein secondary structure determination or gene finding relies on discovering characteristic statistical properties of protein sequences. Name a property (and justify your answer) that helps to localise (and distinguish) transmembrane segments and coils in a protein sequence, or exon/intron boundaries in a genomic region. [2 marks]

(b) Discuss the complexity of an algorithm to reconstruct a genetic network from microarray perturbation data. [7 marks]

(c) What is the difference in terms of connectivity between a scale-free network and a random network? Give biological examples of scale-free networks. [3 marks]

11 Software Engineering and Design

(a) Describe, with simple illustrative drawings, four of the diagrams in the Unified Modeling Language (UML). [3 marks each]

(b) How does the use of each of these diagrams map onto the spiral development approach to software engineering? [2 marks each]
12 Complexity Theory

(a) Suppose that you were provided with a black box that could accept the language of sentences describing an integer $k$ and a graph $G$ with a $k$-clique, and the black box accepted such languages in polynomial time. Explain how you could derive a process that would accept satisfiable instances of the problem 3-SAT in polynomial time. [10 marks]

(b) Suppose instead you had been provided with a black box that provided a polynomial-time acceptor for 3-SAT. Explain how you could use that to derive an efficient acceptor for the clique problem. [10 marks]

In your explanation the level of detail you are expected to give should be tuned to the level of complication in any transformations that you need to describe: simple ones should be described and justified in detail while elaborate or messy ones can be sketched and standard results quoted.

END OF PAPER