Numerical Analysis II

(a) Explain the term positive semi-definite. If $A$ is a real square matrix show that $A^T A$ is symmetric and positive semi-definite. [3 marks]

(b) How is the $l_2$ norm of $A$ defined? State Schwarz’s inequality for the product $Ax$. [2 marks]

(c) Describe briefly the properties of the matrices $U$, $W$, $V$ in the singular value decomposition $A = UWV^T$. [3 marks]

(d) Let $\hat{x}$ be an approximate solution of $Ax = b$, and write $r = b - A\hat{x}$, $e = x - \hat{x}$. Derive a computable estimate of the relative error $\|e\|/\|x\|$ in the approximate solution, and show how this may be used with the $l_2$ norm. [8 marks]

(e) Suppose $A$ is a $7 \times 7$ matrix whose singular values are $10^2$, $10^{-4}$, $10^{-10}$, $10^{-16}$, $10^{-22}$, $10^{-29}$, $10^{-56}$. Construct the matrix $W^+$ that you would use (i) if machine epsilon $\approx 10^{-15}$, and (ii) if machine epsilon $\approx 10^{-30}$. [4 marks]