2005 Paper 8 Question 10

Information Theory and Coding

(a) For continuous random variables X and Y, taking on continuous values x and y respectively with probability densities $p(x)$ and $p(y)$ and with joint probability distribution $p(x, y)$ and conditional probability distribution $p(x|y)$, define:

(i) the *differential entropy* $h(X)$ of random variable X; [1 mark]

(ii) the *joint entropy* $h(X, Y)$ of the random variables X and Y; [1 mark]

(iii) the *conditional entropy* $h(X|Y)$ of X, given Y; [1 mark]

(iv) the *mutual information* $i(X; Y)$ between the continuous random variables X and Y; [1 mark]

(v) how the *channel capacity* of a continuous channel which takes X as its input and emits Y as its output would be determined. [1 mark]

(b) For a time-varying continuous signal $g(t)$ which has Fourier transform $G(k)$, state the *modulation theorem* and explain its rôle in AM radio broadcasting. How does modulation enable many independent signals to be encoded into a common medium for transmission, and then separated out again via tuners upon reception? [4 marks]

(c) Briefly define

(i) The *Differentiation Theorem* of Fourier analysis: if a function $g(x)$ has Fourier transform $G(k)$, then what is the Fourier transform of the n^{th} derivative of $g(x)$, denoted $g^{(n)}(x)$? [2 marks]

(ii) If discrete symbols from an alphabet $\mathcal{S}$ having entropy $H(\mathcal{S})$ are encoded into blocks of length n, we derive a new alphabet of symbol blocks $\mathcal{S}^n$. If the occurrence of symbols is independent, then what is the entropy $H(\mathcal{S}^n)$ of the new alphabet of symbol blocks? [2 marks]

(iii) If symbols from an alphabet of entropy H are encoded with a *code rate* of R bits per symbol, what is the *efficiency* η of this coding? [2 marks]

(d) Briefly explain

(i) how 10 V is expressed in dBμV; [1 mark]

(ii) the YCrCb coordinate system. [4 marks]