Types

(a) State the value-restricted typing rule for let-expressions in ML. [5 marks]

(b) Which of the following typing judgements are provable in the ML type system with the value-restricted rule for let-expressions? Justify your answer in each case, stating any other of the ML typing rules that you use. For part (iii) you must decide whether or not there is a type scheme $\sigma$ that makes the typing provable.

(i) $\{\} \vdash \text{let } r = \text{ref } \lambda x (x) \text{ in } (!r)(r := \lambda y (\text{true})) : \text{unit}$ [5 marks]

(ii) $\{\} \vdash \text{let } r = \text{ref } \lambda x (x) \text{ in } (!r)(r := \lambda y (())) : \text{unit}$ [3 marks]

(iii) $\{\} \vdash \text{let } f = \lambda x (\text{ref } x) \text{ in } f f : \sigma$ [3 marks]

(iv) $\{x : \alpha\} \vdash \lambda f (f x) : \forall \beta((\alpha \to \beta) \to \beta)$ (where $\alpha$ and $\beta$ are distinct type variables) [2 marks]

(v) $\{x : \beta\} \vdash \lambda f (f x) : \forall \beta((\beta \to \beta) \to \beta)$ [2 marks]