

2005 Paper 7 Question 8

Information Theory and Coding

- (a) For a binary symmetric communication channel whose input source is the alphabet $X = \{0, 1\}$ with probabilities $\{0.5, 0.5\}$ and whose output alphabet is $Y = \{0, 1\}$, having the following channel matrix where ϵ is the probability of transmission error:

$$\begin{pmatrix} 1 - \epsilon & \epsilon \\ \epsilon & 1 - \epsilon \end{pmatrix}$$

- (i) How much uncertainty is there about the input symbol once an output symbol has been received? [5 marks]
- (ii) What is the mutual information $I(X; Y)$ of this channel? [2 marks]
- (iii) What value of ϵ maximises the uncertainty $H(X|Y)$ about the input symbol given an output symbol? [1 mark]
- (b) For a continuous (i.e. non-discrete) function $g(x)$, define:
- (i) its continuous Fourier transform $G(k)$; [2 marks]
- (ii) the inverse Fourier transform that recovers $g(x)$ from $G(k)$. [2 marks]
- (c) What simplifications occur in the Fourier representation of a function if:
- (i) the function is real-valued rather than complex-valued? [1 mark]
- (ii) the function has even symmetry? [1 mark]
- (iii) the function has odd symmetry? [1 mark]
- (d) Give a bit-string representation of the number 13 in
- (i) unary code for non-negative integers; [1 mark]
- (ii) Golomb code for non-negative integers with parameter $b = 3$; [2 marks]
- (iii) Elias gamma code for positive integers. [2 marks]