2005 Paper 7 Question 5

Computer Systems Modelling

Consider the s server $M/M/s$ queue with arrival rate λ and service rate μ.

(a) Consider the birth–death process modelling the number of customers present and sketch the state space diagram showing the states of the system, the possible transitions between the states and the transition rates.   [4 marks]

(b) Write down the detailed balance equations for this birth–death process.
[4 marks]

(c) Find the equilibrium distribution, π_n ($n = 0, 1, \ldots$), for the number of customers present by solving the detailed balance equations in terms of π_0.
[6 marks]

(d) By normalising these expressions for π_n, find an expression for π_0 and hence show that the equilibrium distribution exists if and only if $\lambda < s\mu$. [6 marks]