(a) Give a lambda expression that can be used to form the composition of two functions. [1 mark]

(b) Suppose that the lambda expression you have given above can be referred to using the name B. One way of representing the natural numbers as lambda expressions involves for instance having the number “3” represented by a term \( \lambda f. B f (B f f) \) so that a numeral when applied to an argument \( f \) composes \( f \) with itself the given number of times.

In this scheme, write out lambda expressions that will serve as 0, 1 and 2. [3 marks]

(c) Present and explain lambda expressions that find the successor to a number represented as in part (b) and that add two numbers together. [6 marks]

(d) If \( m \) and \( n \) are two lambda expressions that both represent numbers in this style, what interpretation can be placed on the term \( (m \ n) \)? Explain and justify your claim. [4 marks]

(e) Explain how it is possible to produce a lambda expression that, given the representation of a non-zero number \( k \), produces an expression that behaves like \( k - 1 \). [6 marks]