2005 Paper 2 Question 5

Probability

By repeatedly dropping a knitting needle onto a floor constructed from parallel-sided floor boards one may estimate the value of π.

Suppose the needle has unit length and the boards have unit width. After each drop the position of the needle may be specified by the values of two independent random variables:

> θ is the (acute) angle that the needle makes with the joints of the boards and it may be assumed that θ is distributed Uniform$(0, \frac{\pi}{2})$. [It is not necessary to consider values of θ in the other three quadrants.]
>
> y is the distance of the centre of the needle from the nearest joint between two boards and it may be assumed that y is distributed Uniform$(-\frac{1}{2}, +\frac{1}{2})$.

(a) Sketch sufficient of the θ–y plane to show the whole of the region R which corresponds to the needle crossing a joint. Deem such a crossing to be a *hit* and show that the probability of a *hit* is $2/\pi$ and, therefore, the value of π may be estimated experimentally by the formula $2 \times$ *drops*/*hits*. [8 marks]

(b) Suppose the needle is dropped n times. Let X be a random variable whose value is the number of *hits*. X is distributed Binomial$(n, 2/\pi)$. Give expressions for the Expectation $\mathrm{E}(X)$ and the Variance $\mathrm{V}(X)$. [2 marks]

(c) Let $\pi_e(n)$ denote the estimated value of π when the number of *hits* is two standard deviations *below* the mean. Show that for

$$\pi_e(n) < \pi + 0.1$$

to hold, the number of *drops* n must be such that

$$n > \frac{2(\pi - 2)(\pi + 0.1)^2}{0.01}$$

[10 marks]