2005 Paper 1 Question 8

Discrete Mathematics

(*a*) (*i*) Define the terms *injection*, *surjection* and *bijection*. [2 marks]

(*ii*) Let A, B and C be sets. Define a bijection from $[(A \times B) \to C]$ to $[A \to (B \to C)]$. Define its inverse. [2 marks]

(*b*) Prove that

(*i*) if A and B are countable sets, then their product $A \times B$ is also countable; [2 marks]

(*ii*) if A and B are countable sets, then their union $A \cup B$ is countable; [2 marks]

(*iii*) the powerset of the natural numbers $\mathcal{P}(\mathbb{N})$ is uncountable; [5 marks]

(*iv*) the set of finite subsets of $\mathbb{N}$ is countable. [2 marks]

[You may assume that a set B is countable iff there is an injection from B into $\mathbb{N}$.]

(*c*) Explain why assuming that the collection $\{x \mid x \text{ is a set}\}$ is a set would lead to a contradiction. [3 marks]

(*d*) By considering the set $B = \mathbb{R} \cup \{b\}$, where $b \notin \mathbb{R}$, define a well-founded relation $\prec$ on B such that $\{x \in B \mid x \prec b\}$ is uncountable. [2 marks]