Foundations of Computer Science

Consider a datatype of binary trees where both leaves and branches carry labels:

\[
\text{datatype } 'a \text{ tree} = \text{Twig of } 'a | \text{Br of } 'a \times 'a \text{ tree} \times 'a \text{ tree};
\]

A path in a binary tree is a series of labels proceeding from the root to a leaf, as shown in the diagram:

Consider the problem of finding a path in a binary tree such that the integer sum of the labels satisfies a given property. (In the example above, the highlighted path sums to a prime number.)

(a) Write an ML function \texttt{find\_path} such that \texttt{find\_path p t} returns some path in t whose sum satisfies the boolean-valued function \texttt{p}. If no such path exists, the function should raise an exception. [5 marks]

(b) Write an ML function \texttt{all\_paths} such that \texttt{all\_paths p t} returns the list of all paths in \texttt{t} whose sums satisfy the boolean-valued function \texttt{p}. [6 marks]

(c) Write an ML function \texttt{all\_pathq} that is analogous to \texttt{all\_paths} but returns a lazy list of paths. For full credit, your function should find paths upon demand rather than all at once. [Hint: try adding solutions to an accumulating argument.] [9 marks]