Numerical Analysis II

(a) Explain the term *positive semi-definite*. If $A$ is a real square matrix show that $A^T A$ is symmetric and positive semi-definite. [3 marks]

(b) How is the $l_2$ norm of $A$ defined? State Schwarz’s inequality for the product $A x$. [2 marks]

(c) Describe briefly the properties of the matrices $U$, $W$, $V$ in the *singular value decomposition* $A = U W V^T$. [3 marks]

(d) Let $\hat{x}$ be an approximate solution of $A x = b$, and write $r = b - A \hat{x}$, $e = x - \hat{x}$. Derive a computable estimate of the relative error $\|e\|/\|x\|$ in the approximate solution, and show how this may be used with the $l_2$ norm. [8 marks]

(e) Suppose $A$ is a $7 \times 7$ matrix whose singular values are $10^2$, $10^{-4}$, $10^{-10}$, $10^{-16}$, $10^{-22}$, $10^{-29}$, $10^{-56}$. Construct the matrix $W^+$ that you would use

(i) if *machine epsilon* $\simeq 10^{-15}$, and

(ii) if *machine epsilon* $\simeq 10^{-30}$. [4 marks]