Data Structures and Algorithms

(a) Explain how a Boolean matrix can be used to represent the edges of a finite directed graph whose vertices are numbered 1 to $n$. [2 marks]

(b) Describe Warshall’s algorithm to convert the matrix representing a graph to one that represents its transitive closure, and carefully explain why the algorithm works. [6 marks]

(c) Outline Floyd’s algorithm, without proof of correctness, to find the cost of the cheapest path between any two vertices of a directed graph where the edges carry non-negative costs. [4 marks]

(d) It is required to construct a matrix $R$ that encodes a path with the minimum number of edges from any vertex $i$ to any other vertex $j$. $R_{ij}$ will be zero if no path exists from vertex $i$ to vertex $j$; otherwise, $R_{ij}$ will hold the vertex number of the next vertex of a minimal path from $i$ to $j$. Suggest an algorithm to compute $R$ from a given Boolean matrix $M$. [8 marks]