Mathematics for Computation Theory

(a) Let $M$ be an $n$-state deterministic finite automaton over the finite alphabet $S$. Write $l(w)$ for the length of words $w \in S^*$. Suppose that $M$ accepts the word $x \in S^*$, where $l(x) \geq n$.

(i) Show that $x$ is a concatenation of words $uvw$, where $l(uv) \leq n$, $l(v) \geq 1$, and $M$ accepts the word $z_k = uv^k w$ for all natural numbers $k \geq 0$.

(ii) Hence show that if $M$ accepts some word $y \in S^*$, it must accept some word $z \in S^*$ such that $l(z) < n$; and that $M$ accepts an infinite set of words if and only if it accepts some word $x \in S^*$ such that $n \leq l(x) < 2n$.

[8 marks]

(b) Let $S = \{a, b\}$ be an alphabet of two symbols. Explain whether each of the following languages over $S$ is regular:

(i) $L_1 = \{uv \mid u, v \in S^*, \quad l(v) = 2l(u)\}$

(ii) $L_2 = \{ww \mid w \in S^*\}$

[3 marks] [4 marks]