COMPUTER SCIENCE TRIPOS Part II (General)
DIPLOMA IN COMPUTER SCIENCE

Tuesday 7 June 2005 1.30 to 4.30

Paper 11 (Paper 2 of Diploma in Computer Science)

Answer **five** *questions.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script Paper
Blue Coversheets
Tags

1 Digital Electronics

(a) A Moore machine is required which produces the counting sequence 0,1,2,3,4,5,0. Give a minimum sum-of-products for each of the next state variables for an implementation of this Moore machine. [6 marks]

(b) Design a two-bit Gray code counter which produces the binary sequence 00,01,11,10,00. The counter should be designed as a Moore machine consisting of D flip-flops (with enable inputs) and a minimal number of logic gates. An additional input (`E`) is required to enable or disable counting which can be connected directly to the enable inputs of the D flip-flops. What is the final circuit diagram? [6 marks]

(c) The 0→5 and Gray-code counters are coupled together to produce a state machine with following state sequence and output pattern in Morse code for SOS ($\cdots\ ---\ \cdots$):

State sequence		Output
00	000	0
00	001	1
00	010	0
00	011	1
00	100	0
00	101	1
01	000	0
01	001	1
01	010	1
01	011	1
01	100	0
01	101	1
11	000	1
11	001	1
11	010	0
11	011	1
11	100	1
11	101	1
10	000	0
10	001	1
10	010	0
10	011	1
10	100	0
10	101	1

With the aid of a circuit diagram, explain how the two counters are coupled together to produce the sequencer, and how the required Morse code output can be generated from this sequencer. [8 marks]

2 Foundations of Programming

(*a*) List the three kinds of Java *inner class* and say which of the three is used in the following program. [4 marks]

```
public class Trial
 { private boolean b = true;

   private Trial()
    { (new ClassIn()).test();
      this.b = false;
    }

   public static void main(String args [])
    { new Trial();
    }

   private class ClassIn
    { ClassIn()
       { boolean b = false;
       }

      private void test()
       { if (this.b)
            System.out.println("Yes");
         else
            System.out.println("No");
       }
    }
}
```

(*b*) The `javac` compiler complains about the condition in the `if`-statement. What is the complaint? Show *two* ways in which the condition may be fixed. [4 marks]

(*c*) With either of these corrections the program will compile and run in the same way. Explain what happens at run-time and, in particular, show what output is printed. [6 marks]

(*d*) Rearrange the code so that `class ClassIn` is a local class within the constructor `Trial()`. The program as a whole should perform as before. [6 marks]

3 Compiler Construction

(*a*) A (phrase-structured) grammar is often defined to be a 4-tuple (N, T, R, S) where R is a set of production rules. Explain what the other components of the 4-tuple are. Explain also the (most general form of) production rules, how these are conventionally restricted and why one might wish to restrict them. [6 marks]

(*b*) Give a grammar which is ambiguous. [2 marks]

(*c*) Give a grammar which is not a regular grammar but which generates a regular language containing an infinite number of strings. [2 marks]

(*d*) Is it possible to write a grammar which generates the strings $\{aa, aaa, aaaaa, \ldots, a^p, \ldots\}$ where p is prime? (A general argument for or against suffices.) [2 marks]

(*e*) It is desired to construct a simple "pocket-calculator" program using yacc and lex (or other similar automated tools of your choice) which can parse strings such as "`1+(10-5-3)*5+2=`" and print the result, 13 in this case. Outline the overall structure of your program components. Give full details of the input to yacc and lex (or equivalent). (Precise syntactic details are not important, but your answer should show an understanding of the principles involved.) [8 marks]

4 Data Structures and Algorithms

(*a*) A closed hash table is one in which the overflow chains of key–value pairs are held within the table itself. Carefully describe how the closed hash table mechanism works for both insertion and lookup. [6 marks]

(*b*) Assume that the initial probe is $p_0 =$ `Hash1(key)` `mod` B and the secondary probes are $p_i, i = 1 \ldots B-1$. Discuss the relative merits of the following schemes for choosing the secondary probes.

(*i*) $p_i = (p_0 + i)$ `mod` B

(*ii*) $p_i = (p_0 + 13 * i)$ `mod` B

(*iii*) $p_i = (p_0 + 13 * i + 17 * i * i)$ `mod` B

(*iv*) $p_i = (p_0 +$ `Hash2(key)` $* i + 17 * i * i)$ `mod` B

You may assume that all the arithmetic is unsigned. [8 marks]

(*c*) Carefully describe a mechanism for deleting key–value pairs from a closed hash table. [6 marks]

5 Artificial Intelligence I

A *perceptron* computes the function $h(\mathbf{x}) = \text{sgn}(\mathbf{w}^T\mathbf{x} + w_0)$ where $\text{sgn}(x) = +1$ if $x \geq 0$ and $\text{sgn}(x) = -1$ otherwise. The *primal perceptron algorithm* is as follows:

```
do
{
   for (each example in s)
   {
      if (y_i(w^T x_i + w_0) ≤ 0)
      {
         w = w + η y_i x_i
         w_0 = w_0 + η y_i R^2
      }
   }
}
while (mistakes are made in the for loop)
```

where η is a positive real, $R = \max_i ||\mathbf{x}_i||$ and $\mathbf{w}$ and w_0 are initialised to be the zero vector and 0 respectively, and where

$$\mathbf{s} = ((\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_m, y_m)) \text{ with } y_i \in \{+1, -1\}$$

is a training sequence.

(*a*) Derive the dual form of the perceptron algorithm and state the corresponding alternative representation for $\mathbf{w}$. [6 marks]

(*b*) Explain how the perceptron may be applied to problems that are not linearly separable by introducing *basis functions*. [4 marks]

(*c*) Give a definition of a *kernel*. [3 marks]

(*d*) Explain how the use of a suitable kernel in conjunction with the dual form of the perceptron algorithm can be advantageous compared with the direct use of basis functions and the primal perceptron algorithm. [7 marks]

6 Operating System Foundations

(*a*) A system has paging hardware but no segmentation hardware. Discuss the likely structure of a process page table. What information would you expect to be held in a page table entry? [5 marks]

(*b*) Describe the operation of the following hardware support options for paging:

(*i*) A pair of processor registers.
PTBR holds the address of the base of the page-table of the current process.
PTLR holds the size of the page table in bytes. [5 marks]

(*ii*) A TLB (Translation Lookaside Buffer). [10 marks]

7 Continuous Mathematics

(a) Let $f(x)$ be a periodic function of period 2π. Give expressions for the Fourier coefficients a_r ($r = 0, 1, \ldots$) and b_r ($r = 1, 2, \ldots$) of $f(x)$ where

$$\frac{a_0}{2} + \sum_{r=1}^{\infty} (a_r \cos rx + b_r \sin rx)$$

is the Fourier series representation of $f(x)$. [2 marks]

(b) Show that the Fourier series in part (a) can also be written as a complex Fourier series

$$\sum_{r=-\infty}^{r=\infty} c_r e^{irx}$$

by deriving expressions for the complex Fourier coefficients c_r ($r = 0, \pm 1, \pm 2, \ldots$) in terms of a_r and b_r. [3 marks]

(c) Use your expressions for a_r and b_r in part (a) and for c_r in part (b) to show that

$$c_r = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-irx} dx \qquad (r = 0, \pm 1, \pm 2, \ldots) .$$

[3 marks]

(d) Show that the complex Fourier coefficients of $f(x - \alpha)$ (where α is a constant) are given by $e^{-ir\alpha} c_r$ ($r = 0, \pm 1, \pm 2, \ldots$). [6 marks]

(e) Suppose that $g(x)$ is another periodic function of period 2π with complex Fourier coefficients d_r ($r = 0, \pm 1, \pm 2, \ldots$) and define $h(x)$ by

$$h(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x - y) g(y) dy .$$

Show that $h(x)$ is a periodic function of period 2π and that its complex Fourier coefficients are given by $h_r = c_r d_r$ ($r = 0, \pm 1, \pm 2, \ldots$). [6 marks]

[*You may assume that the periodic functions in this question satisfy the Dirichlet conditions. Euler's equation may be used without proof but should be stated precisely.*]

8 Numerical Analysis I

(*a*) Define *absolute error* and *relative error*. How are these related? Explain briefly the term *loss of significance*. [3 marks]

(*b*) An algorithm is required for solution of $ax^2 + bx + c = 0$ where $b > 0$. Describe how loss of significance can occur in the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and derive an alternative formula for such a case. Illustrate your answer by applying it to the case $a = 30$, $b = 3000$, $c = 1$ on a decimal machine with only 5 significant digits available. [You should assume for the purposes of calculation that $\sqrt{b^2 - 4ac}$ evaluates to $b - (2ac/b)$, correctly rounded.] [10 marks]

(*c*) The series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

is to be summed by taking terms in order, left to right, using only p decimal digits of precision until additional terms are negligible. If $x = 6$ find the largest term of the series and hence, assuming $\cos 6 \simeq 1$, estimate roughly how many decimal digits of accuracy will be lost in the process. [7 marks]

9 Mathematics for Computation Theory

(a) Let M be an n-state deterministic finite automaton over the finite alphabet S. Write $l(w)$ for the length of words $w \in S^*$. Suppose that M accepts the word $x \in S^*$, where $l(x) \geqslant n$.

(i) Show that x is a concatenation of words uvw, where $l(uv) \leqslant n$, $l(v) \geqslant 1$, and M accepts the word $z_k = uv^k w$ for all natural numbers $k \geqslant 0$.
[8 marks]

(ii) Hence show that if M accepts some word $y \in S^*$, it must accept some word $z \in S^*$ such that $l(z) < n$; and that M accepts an infinite set of words if and only if it accepts some word $x \in S^*$ such that $n \leqslant l(x) < 2n$.
[5 marks]

(b) Let $S = \{a, b\}$ be an alphabet of two symbols. Explain whether each of the following languages over S is regular:

(i) $L_1 = \{uv \mid u, v \in S^*, \quad l(v) = 2.l(u)\}$ [3 marks]

(ii) $L_2 = \{ww \mid w \in S^*\}$ [4 marks]

10 Computation Theory

(a) What does it mean for a subset S of the set $\mathbb{N}$ of natural numbers to be register machine *decidable*? [3 marks]

(b) For each $e \in \mathbb{N}$, let $\varphi_e \in \mathit{Pfn}(\mathbb{N}, \mathbb{N})$ denote the partial function computed by the register machine program with index e. Let $e_0 \in \mathbb{N}$ be an index for the totally undefined partial function (so that $\varphi_{e_0}(x)\uparrow$, for all $x \in \mathbb{N}$).

Suppose that a total function $f \in \mathit{Fun}(\mathbb{N}, \mathbb{N})$ is *extensional*, in the sense that for all $e, e' \in \mathbb{N}$, $f(e) = f(e')$ if φ_e and $\varphi_{e'}$ are equal partial functions. Suppose also that the set $S_f = \{x \in \mathbb{N} \mid f(x) = f(e_0)\}$ is not the whole of $\mathbb{N}$, so that for some $e_1 \in \mathbb{N}$, $f(e_1) \neq f(e_0)$.

(i) If membership of S_f were decided by a register machine M, show informally how to construct from M a register machine M' that, started with $R1 = e$ and $R2 = n$ (any $e, n \in \mathbb{N}$) always halts, with $R0 = 0$ if $\varphi_e(n)\downarrow$ and with $R0 = 1$ if $\varphi_e(n)\uparrow$. Make clear in your argument where you use the fact that f is extensional.

[Hint: For each $e, n \in \mathbb{N}$ consider the index $i(e, n) \in \mathbb{N}$ of the register machine that inputs x, computes $\varphi_e(n)$ and if that computation halts, then computes $\varphi_{e_1}(x)$.] [14 marks]

(ii) Deduce that if f is extensional, then S_f is either the whole of $\mathbb{N}$, or not decidable. [3 marks]

11 Software Engineering and Design

Imagine that you are the software designer responsible for a system that manages the shutdown of a nuclear power station. In normal shutdown mode fuel rods are withdrawn immediately, but steam generation is stopped only after turbine speed has dropped to one quarter. However, in emergency shutdown mode steam generation must be stopped immediately.

(a) Sketch initial UML diagrams showing important aspects of your design, including a use case diagram, a class diagram, a sequence diagram and a statechart diagram. [4 marks each]

(b) What international quality standards should be applied in managing these and subsequent design documents? What procedures would result? [4 marks]

12 Business Studies

(*a*) Explain the difference between *debt* and *equity* financing. [5 marks]

(*b*) What is meant by a *convertible debenture with coupon*, and how does this differ from a *redeemable preference share*? [5 marks]

(*c*) A certain software company has assets valued at £1M and is making a net profit of £100K on a turnover of £1M growing at 10% per annum. 1m shares, including 100K staff options, have been issued. No dividends have been paid. The company wish to raise money so that they can expand faster, and are prepared to sell up to 300K new shares. Price the issue. [5 marks]

(*d*) Why do venture capital funds prefer to invest in preference shares? [5 marks]

END OF PAPER