COMPUTER SCIENCE TRIPOS  Part II (General)  
DIPLOMA IN COMPUTER SCIENCE  

Monday 6 June 2005  1.30 to 4.30

Paper 10 (Paper 1 of Diploma in Computer Science)

Answer five questions.

Submit the answers in five separate bundles, each with its own cover sheet. On each cover sheet, write the numbers of all attempted questions, and circle the number of the question attached.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script Paper
Blue Coversheets
Tags
1 Digital Electronics

(a) What is a minimum sum-of-products? [3 marks]

(b) A full adder has data inputs ($A_0, B_0$) and a carry input ($C_0$). The sum ($S_0$) and carry ($C_1$) are output. What are the minimum sum-of-products equations for $S_0$ and $C_1$? [6 marks]

(c) How could the gate count for the implementation of output $S_0$ be reduced using XOR gates? [2 marks]

(d) For a 3-bit full adder (i.e. one which has three $A$ inputs ($A_0, A_1, A_2$), three $B$ inputs ($B_0, B_1, B_2$) and three sum outputs ($S_0, S_1, S_2$)), the final carry output is $C_3$. What is the sum-of-products equation for $C_3$ in terms of the $A$ and $B$ inputs? [6 marks]

(e) If we were to implement an 8-bit full adder, why would we look for a multi-level logic implementation for the carry output ($C_8$)? [3 marks]
2 Foundations of Programming

(a) Distinguish between the terms instance method and class method. [4 marks]

(b) A newcomer to Java programming has written the following code:

```java
class Parent {
    public void test() {
        System.out.println("Parent");
    }
}

class Child extends Parent {
    public static void main(String[] args) {
        Parent p = new Parent();
        Child c = new Child();
        p.test();
        c.test();

        p = c;
        p.test();
        c.test();

        c = p;
        p.test();
        c.test();
    }

    public void test() {
        System.out.println("Child");
    }
}
```

The `javac` compiler complains about one statement. Which one and why? Correct the code by inserting an appropriate cast. [4 marks]

(c) With this correction the program will compile and run. Explain in outline what happens at run-time and show what output is printed. [5 marks]

(d) Small print in the Java documentation says that you “cannot override a static method but you can hide it”. If both `test()` methods are made static the program will again compile and run. Explain what happens this time and show what output is printed. [7 marks]
3 Data Structures and Algorithms

(a) Briefly outline how a sequence of symbols can be encoded as a sequence of Huffman codes, and explain under what assumptions Huffman encoding generates optimally compact code. [6 marks]

(b) Estimate the number of bits needed to Huffman encode a random permutation of As, Bs and Cs, with each letter occurring one million times. [3 marks]

(c) Estimate the number of bits needed to Huffman encode a random permutation of As, Bs and Cs, where A occurs two million times and B and C each occur one million times. [3 marks]

(d) Estimate how many bits would be needed to encode the sequence in part (b) above using arithmetic coding. You may assume that $\log_2 3$ is about 1.6. [4 marks]

(e) Estimate, with justification, how many bits would be needed to encode the sequence in part (c) above using arithmetic coding. [4 marks]

4 Artificial Intelligence I

(a) What are the advantages and disadvantages of constraint satisfaction problem (CSP) solvers compared with search algorithms such as A* search, etc? [3 marks]

(b) Give a general definition of a CSP. Define the way in which a solution is represented and what it means for a solution to be consistent and complete. [5 marks]

(c) Assuming discrete binary constraints and finite domains, explain how breadth-first-search might be used to find a solution and why this is an undesirable approach. [3 marks]

(d) Give a brief description of the basic backtracking algorithm for finding a solution. [4 marks]

(e) Describe the minimum remaining values heuristic, the degree heuristic and the least constraining value heuristic. [5 marks]
5 Comparative Programming Languages

Consider the Prolog procedures named \( s \) and \( p \) defined as follows:

\[
\begin{align*}
    s(H, [H|T], T). \\
    s(H, [N|T], [N|L]) & : - s(H, T, L). \\
    p(X, [H|T]) & : - s(H, X, Z), p(Z, T). \\
    p([], []). 
\end{align*}
\]

\((a)\) Show how Prolog would evaluate the goal \( s(H, \{a,b,c\}, T) \) giving all the successive instantiations of \( H \) and \( T \) that cause the goal to be satisfied, and hence describe in words what \( s \) does. \([6\text{ marks}]\)

\((b)\) What value of \( Q \) causes the goal \( p([a], Q) \) to be satisfied? \([3\text{ marks}]\)

\((c)\) What values of \( Q \) cause the goal \( p([a,b], Q) \) to be satisfied? \([4\text{ marks}]\)

\((d)\) What values of \( Q \) cause the goal \( p([a,b,c], Q) \) to be satisfied? \([5\text{ marks}]\)

\((e)\) Describe in words what \( p \) does. \([2\text{ marks}]\)
6 Operating System Foundations

(a) A device driver process carries out character I/O via a Universal Asynchronous Receiver/Transmitter (UART).

(i) Why is hardware–software synchronisation needed? [1 mark]

(ii) Describe polled operation. [2 marks]

(iii) Describe interrupt-driven operation. [2 marks]

(iv) Draw a state transition diagram for the device-driver process. Indicate the events that cause each transition and in each case explain the effect on the device driver’s process descriptor and the operating system’s scheduling queues. Assume interrupt-driven software. [7 marks]

(b) The device driver process fills/empties a buffer of fixed size in an I/O buffer area. A process carrying out application requests reads and writes data in variable-sized amounts from the buffer.

(i) Why must mutually exclusive access to the buffer be enforced? [2 marks]

(ii) Why is condition synchronisation needed? [2 marks]

(iii) What is wrong with the following pseudocode fragment from the device-driver’s specification, where:

- `buffer-lock` is a semaphore initialised to 1,
- `space` is a semaphore initialised to the buffer size in bytes,
- `data` is a semaphore initialised to 0?

```
on input:
WAIT(buffer-lock);
if buffer is full then WAIT(space);
write a character into the buffer;
SIGNAL(buffer-lock);

on output:
WAIT(buffer-lock);
if buffer is empty then WAIT(data);
read a character from the buffer;
SIGNAL(buffer-lock);
```

[4 marks]
7 Numerical Analysis I

(a) The parameters for IEEE Single Precision are: \( \beta = 2 \), \( p = 24 \), \( e_{\text{min}} = -126 \), \( e_{\text{max}} = 127 \). Explain the terms significand, sign bit, exponent, normalised number, denormal number, hidden bit, precision as used in IEEE Single Precision. [7 marks]

(b) Let \( \omega \) represent any of the operations + − ∗ /. Let \( x \) be a positive finite representable number. List what each of the following evaluates to for each operation:

\[
\begin{align*}
(+\infty) \; \omega \; x \\
x \; \omega \; (-\infty)
\end{align*}
\]

[Show the sign of your answer in each case.] [4 marks]

(c) Suppose the principles of IEEE arithmetic are applied to a floating-point representation with 6 bytes (48 stored bits). If \( \beta = 2 \), \( e_{\text{max}} = 511 \) and a hidden bit is used, deduce the values of \( e_{\text{min}} \) and \( p \). [4 marks]

(d) Define machine epsilon \( \varepsilon_m \). [1 mark]

(e) The function

\[
f(x) = \frac{(x + 1)^2}{x^2 + 1}
\]

is to be evaluated using IEEE arithmetic for \( x \geq 0 \). Re-write the formula so that \( f(x) \) can be evaluated in the case where \( x \) is representable but \( x^2 \) overflows. What does your formula evaluate to in the case that \( 1/x < \varepsilon_m \)? [4 marks]
8 Mathematics for Computation Theory

State the requirements for \((S, \leq)\) to be:

(a) a partially ordered set;

(b) a totally ordered set;

(c) a well ordered set. [5 marks]

Let \((\mathbb{N}, \leq)\) be the natural numbers under the standard ordering. Define the product ordering \(\leq_p\) on \((\mathbb{N} \times \mathbb{N})\) that is derived from this ordering. Which of conditions (a), (b), (c) does \(\leq_p\) satisfy? [3 marks]

Let \((S, \leq)\) and \((T, \prec)\) be partially ordered sets, and \(f : (S, \leq) \to (T, \prec)\) be a function. What condition must be satisfied in order that \(f\) be monotonic? [2 marks]

If \(f\) is a bijection, and both \(f\) and \(f^{-1}\) are monotonic, we say that \((S, \leq), (T, \prec)\) are isomorphic partially ordered sets.

Suppose that \((S, \leq)\) is a partially ordered set. A topological sort of \((S, \leq)\) is defined by specifying a total ordering \(\sqsubseteq\) on \(S\) such that the identity map \(i : (S, \leq) \to (S, \sqsubseteq)\) is monotonic.

Define two different topological sorts of \((\mathbb{N} \times \mathbb{N}, \leq_p)\), one of which is isomorphic to \(\mathbb{N}\) with the standard ordering, while the other is not. Justify your claims. [10 marks]

9 Computation Theory

(a) Explain informally, i.e. without reference to any particular model of computation, why the Halting Problem is undecidable. [6 marks]

(b) Briefly describe two mathematical problems, other than the Halting Problem, that are algorithmically undecidable. [4 marks]

(c) What does it mean for a partial function to be register machine computable? Show how the informal argument in part (a) can be turned into a rigorous proof that there is no register machine deciding the Halting Problem for register machine computable functions. [10 marks]
10 Computer Graphics and Image Processing

(a) Calculate the maximum resolution needed by a movie projector in a movie theatre. Clearly state any assumptions that you make. [6 marks]

(b) Describe, in detail, an error diffusion algorithm for converting greyscale images to bi-level black and white images at the same resolution. [8 marks]

(c) Explain how this could be extended to provide an algorithm to print full colour RGB images on a CMYK laser printer, assuming that one pixel in the image maps to one pixel on the printer. [6 marks]

11 Introduction to Security

(a) A and B play a simple game. A chooses a number $R_A \in \mathbb{Z}_3$ and B chooses a number $R_B \in \mathbb{Z}_3$. Then A and B communicate their respective choice to each other simultaneously, meaning that the players cannot change their choice after having seen that of the opponent. These rules decide who wins the game:

$$R_A \equiv R_B + 1 \pmod{3} \Rightarrow A \text{ wins}$$

$$R_B \equiv R_A + 1 \pmod{3} \Rightarrow B \text{ wins}$$

In any other case, the result of the game is a draw.

(i) What complication arises when this game is played at a distance, for example via e-mail? [2 marks]

(ii) Suggest a cryptographic protocol that prevents cheating when this game is played via e-mail. Your solution should not rely on a trusted third party. [6 marks]

(iii) What assumptions do you make about the cryptographic functions used in your solution of part (ii)? [3 marks]

(iv) What assumptions do you make about the amount of computing power available to the opponent in your solution of part (ii)? [3 marks]

(b) Outline briefly the purpose of an organisation’s security policy and what steps should be considered in its development. [6 marks]
12 Software Engineering and Design

Imagine that you are the user interface designer responsible for a system that manages the shutdown of a nuclear power station.

(a) Comment on hazards, risks and reliability. [3 marks]

(b) What special procedures should be followed during design? [3 marks]

(c) There is some debate among the team about whether the operator should be in the control loop. What options are there? [4 marks]

(d) In order to assess alternative options like those in part (c):

(i) How could you estimate the speed of operator action based on a draft interface layout? [4 marks]

(ii) How could you measure the speed of operator action using alternative prototypes? [4 marks]

(iii) How could you estimate the probability of operator error? [2 marks]

END OF PAPER