Database Theory

Consider a database with one binary relation $B$.

$(a)$ Write programs in stratified Datalog for the following queries:

$(i)$ Give the set of elements $x$ for which there are fewer than three elements $y$ such that $B(x, y)$. [3 marks]

$(ii)$ Give the set of elements $x$ such that there is no path (in the graph formed by $B$) from $x$ back to itself. [3 marks]

$(iii)$ Give the set of elements $x$ such that $B(x, y)$ for every $y$. [3 marks]

$(b)$ Which of the queries defined in part $(a)$ is:

$(i)$ safe? [3 marks]

$(ii)$ domain independent? [4 marks]

$(iii)$ monotone? [4 marks]

In each case, justify your answer fully.