2004 Paper 9 Question 10

Types

Let β be a type variable and let α range over type variables distinct from β. The subsets of polymorphic lambda calculus (PLC) types that are *positive* (ranged over by τ) and *negative* (ranged over by ν) in β are defined by the following grammar:

$$\tau \ ::= \ \forall \alpha(\tau) \mid \alpha \mid \beta \mid \nu \to \tau$$
$$\nu \ ::= \ \forall \alpha(\nu) \mid \alpha \mid \tau \to \nu$$

(a) Give inductive definitions, following the structure of the grammar above, of closed PLC terms P_τ for each positive type τ, and N_ν for each negative type ν, such that

$$\emptyset \vdash P_\tau : \forall \alpha_1, \alpha_2((\alpha_1 \to \alpha_2) \to (\tau[\alpha_1/\beta] \to \tau[\alpha_2/\beta]))$$
$$\emptyset \vdash N_\nu : \forall \alpha_1, \alpha_2((\alpha_1 \to \alpha_2) \to (\nu[\alpha_2/\beta] \to \nu[\alpha_1/\beta]))$$

[12 marks]

(b) Now let τ be the type $\forall \alpha((\beta \to \alpha) \to \alpha)$, which is positive in β. Calculate the beta-normal form of P_τ. [8 marks]

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