

2004 Paper 9 Question 10

Types

Let β be a type variable and let α range over type variables distinct from β . The subsets of polymorphic lambda calculus (PLC) types that are *positive* (ranged over by τ) and *negative* (ranged over by ν) in β are defined by the following grammar:

$$\begin{aligned}\tau &::= \forall\alpha(\tau) \mid \alpha \mid \beta \mid \nu \rightarrow \tau \\ \nu &::= \forall\alpha(\nu) \mid \alpha \mid \tau \rightarrow \nu\end{aligned}$$

- (a) Give inductive definitions, following the structure of the grammar above, of closed PLC terms P_τ for each positive type τ , and N_ν for each negative type ν , such that

$$\begin{aligned}\emptyset \vdash P_\tau &: \forall\alpha_1, \alpha_2((\alpha_1 \rightarrow \alpha_2) \rightarrow (\tau[\alpha_1/\beta] \rightarrow \tau[\alpha_2/\beta])) \\ \emptyset \vdash N_\nu &: \forall\alpha_1, \alpha_2((\alpha_1 \rightarrow \alpha_2) \rightarrow (\nu[\alpha_2/\beta] \rightarrow \nu[\alpha_1/\beta]))\end{aligned}$$

[12 marks]

- (b) Now let τ be the type $\forall\alpha((\beta \rightarrow \alpha) \rightarrow \alpha)$, which is positive in β . Calculate the beta-normal form of P_τ . [8 marks]