Semantics of Programming Languages

L1 has the expression syntax

\[ e ::= n \mid b \mid e_1 op e_2 \mid \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \mid \ell ::= e \mid !\ell \mid \text{skip} \mid e_1; e_2 \mid \text{while } e_1 \text{ do } e_2 \]

(a) Give the reduction rules for conditionals and while-loops. \([3 \text{ marks}]\)

(b) Define semantic equivalence, \( e_1 \simeq^T_1 e_2 \), for L1. \([4 \text{ marks}]\)

(c) For each of the following pairs, state whether they are semantically equivalent; if not, state a nontrivial condition on the subexpressions \( e, e_1, e_2, e_3 \) that makes them so, and explain informally why it suffices.

\[(i) \quad l := 3; e \simeq^? e; l := 3 \quad \text{[3 marks]}\]

\[(ii) \quad e; (\text{if } e_1 \text{ then } e_2 \text{ else } e_3) \simeq^? \text{if } e_1 \text{ then } e; e_2 \text{ else } e; e_3 \quad \text{[3 marks]}\]

\[(iii) \quad e; (\text{if } e_1 \text{ then } e_2 \text{ else } e_3) \simeq^? \text{if } e; e_1 \text{ then } e_2 \text{ else } e_3 \quad \text{[3 marks]}\]

\[(iv) \quad \text{while } !l \geq 0 \text{ do } (e_2; e_3) \]

\[\simeq^?\]

\[\text{if } !l \geq 0 \text{ then } e_2; (\text{while } !l \geq 0 \text{ do } (e_3; e_2)); e_3 \text{ else skip} \quad \text{[4 marks]}\]