Discrete Mathematics

Suppose that $A$ is a finite set with a bijection: $A \to A \times A$. Calculate $|A|$. [2 marks]

Give an example of a countably infinite set $B$ with a bijection: $B \to B \times B$, proving the result carefully. [4 marks]

Consider the following definitions:

$M = \{n \in \mathbb{N} \mid 2|n\}$, the even numbers
$O = \mathbb{N} \setminus M$, the odd numbers
$P = \mathcal{P}(\mathbb{N})$, the set of subsets of $\mathbb{N}$
$Q = \mathcal{P}(M)$
$R = \mathcal{P}(O)$

Show that $P$, $Q$ and $R$ are uncountable, and construct a bijection: $P \to Q \times R$. [12 marks]

Hence show that there is an uncountable set $C$ with a bijection: $C \to C \times C$. [2 marks]