Foundations of Computer Science

In ML it is possible to use functions as values: they can be passed as arguments and returned as results. Explain the notation used to write a function without having to give it a name. [2 marks]

This question looks at two different ways of implementing functional arrays.

(a) One possible functional implementation of an array is based on trees, and the path to a stored value follows the binary code for the subscript:

```
1
/ \
2   3
/   / \
4   6   5   7
/   /   /   / ...
```

where in the above diagram the numbers show where in the tree a value with the given subscript will live.

Write code that creates, retrieves values from and updates an array that has this representation, and using big-O notation explain the associated costs. [8 marks]

(b) A different way of handling functional arrays is to represent the whole array directly by a function that maps from integers to values. To access the item at position $k$ in such an array you just use the array as a function and give it $k$ as its argument, and to update the array you need to create a new function reflecting the changed value.

If the array is to hold integer values, what ML type does it have? [1 mark]

Write a function `update a n v` where `a` is a functional array in this style, `n` is an integer index and `v` is a new value. The result of the call to `update` must behave as an array that stores all the values that `a` did except that it is as if an assignment of the style “`a[n] := v`” has been performed. [5 marks]

In big-O notation, what is the cost of your `update` function? After a sequence of updates what is the cost of accessing the array? [4 marks]