Complexity Theory

(a) Define a one-way function. [4 marks]

(b) Explain why the existence of one-way functions would imply that $P \neq NP$. [7 marks]

(c) Recall that $Reach$ is the problem of deciding, given a graph $G$ a source vertex $s$ and a target vertex $t$, whether $G$ contains a path from $s$ to $t$; and $Sat$ is the problem of deciding whether a given Boolean formula is satisfiable.

For each of the following statements, state whether it is true or false and justify your answer.

(i) If $Reach$ is NP-complete then $P = NP$. [3 marks]

(ii) If $Reach$ is NP-complete then $NP \neq PSPACE$. [3 marks]

(iii) If $Sat$ is PSPACE-complete then $NP = PSPACE$. [3 marks]