Numerical Analysis II

(a) State a recurrence formula for the sequence of Chebyshev polynomials, 
\(\{T_k(x)\}\), and list these as far as \(T_5(x)\). \[4 \text{ marks}\]

(b) What is the best \(L_\infty\) polynomial approximation over \([-1, 1]\) to \(x^k\) using polynomials of lower degree, and what is its degree? Use this property to explain the method of economisation of a Taylor series. How can the error in one economisation step be estimated? \[7 \text{ marks}\]

(c) It is required to approximate the function \(f(x) = \lim_{k \to \infty} P_k(x)\) over \([-1, 1]\) with an absolute accuracy of 2 decimal places, where

\[
P_k(x) = \sum_{n=1}^{k} \frac{x^n}{n!}.
\]

As this series converges faster than \(e^x\), a good estimate of the error \(\|f(x) - P_k(x)\|_\infty\) in the truncated Taylor series is given by evaluating the next term

\[
\frac{x^{k+1}}{(k + 1)(k + 1)!}
\]

at \(x = 1\). Use the method of economisation to find a polynomial approximation of the required accuracy. \[9 \text{ marks}\]