2004 Paper 11 Question 10

Computation Theory

(a) Explain what is meant by the following statements:

(i) $f : \mathbb{N} \to \mathbb{N}$ is a *total recursive* (TR) function; [3 marks]

(ii) the sequence $\{f_n : \mathbb{N} \to \mathbb{N}\}_{n \in \mathbb{N}}$ of TR functions of a single variable is recursively enumerable. [4 marks]

(b) Show that no recursive enumeration can include the set of *all* TR functions of a single variable. [4 marks]

(c) Suppose $u(n, x)$ is a recursive enumeration of the sequence of TR functions $f_n(x) = u(n, x)$. Show how to define a sequence $\{g_n : \mathbb{N} \to \mathbb{N}\}$ of TR functions of a single variable such that each g_n is distinct from every function f_n, and also from each g_k for $k \neq n$. [5 marks]

(d) Express the sequence $\{g_n\}$ as an explicit recursive enumeration $v(n, x) = g_n(x)$. [4 marks]