Denotational Semantics

(a) State without details three ways to prove properties of the least fixed point $\text{fix}(f)$ of a continuous function $f$ on a domain. [3 marks]

(b) Let $f : D \to D$ be a continuous function on a domain $D$. Explain why $\text{fix}(f \circ f) = \text{fix}(f)$. [3 marks]

(c) Let both $f : D \to D$ and $g : D \to D$ be continuous functions on a domain $D$. Prove

$$\text{fix}(f \circ g) = f(\text{fix}(g \circ f))$$

by showing

(i) $\text{fix}(f \circ g) \sqsubseteq f(\text{fix}(g \circ f))$, and [4 marks]

(ii) $f(\text{fix}(g \circ f)) \sqsubseteq \text{fix}(f \circ g)$. [10 marks]

[Hint: For the last part, start by expanding the left-hand side as a least upper bound of approximations.]