

## 2003 Paper 9 Question 10

### Types

(a) Describe the relation  $=_\beta$  of *beta-conversion* between terms of the polymorphic lambda calculus (PLC). How can one decide whether two typeable PLC terms are in this relation? Why does the decision procedure fail for untypeable terms? [8 marks]

(b) Let  $\omega$  be the polymorphic type  $\forall\alpha_1((\forall\alpha_2(\alpha_2 \rightarrow \alpha_1)) \rightarrow \alpha_1)$ . Show that there is a closed PLC term  $I$  with the following two properties.

(i)  $I$  has type  $\forall\alpha(\alpha \rightarrow \omega)$ .

(ii) If  $M_1$  and  $M_2$  are any closed PLC terms of the same type,  $\tau$  say, and if  $(I \tau M_1) =_\beta (I \tau M_2)$ , then  $M_1 =_\beta M_2$ .

[Hint: for property (ii), consider the beta-normal forms of  $I \tau M_1 \alpha x$  and  $I \tau M_2 \alpha x$ , where  $\alpha$  is a type variable and  $x$  is a variable.] [12 marks]