Computer Systems Modelling

Let $N(t)$ denote the number of events in the time interval $[0, t]$ for a (homogeneous) Poisson process of rate $\lambda$, $(\lambda > 0)$.

(a) State the necessary properties on $N(t)$ that define a (homogeneous) Poisson process of rate $\lambda$. [4 marks]

(b) By dividing the interval $[0, t]$ into equal length sub-intervals show that $N(t)$ is a Poisson random variable with mean $\lambda t$. [4 marks]

(c) Let $X_1$ denote the time of the first event and for $n > 1$ let $X_n$ denote the elapsed time between the $(n-1)$th and the $n$th events of the Poisson process. Determine the distribution of $X_1$ and the joint distribution of $X_1$ and $X_2$. [4 marks]

(d) Let $S_n = \sum_{i=1}^{n} X_i$ denote the time of the $n$th event. Derive the probability density function of the random variable $S_n(t)$. [4 marks]

(e) Give an algorithm to generate the first $T$ time units of a (homogeneous) Poisson process of rate $\lambda$. [4 marks]