Logic and Proof

(a) Define the concepts of a true sequent, a valid sequent and a basic sequent. You may take the concepts of true and valid formulæ as primitive. [3 marks]

(b) Gentzen claimed that his proof systems performed natural logical reasoning. Consider the following sequent calculus rules:

\[
\frac{A, \Gamma \Rightarrow \Delta \quad B, \Gamma \Rightarrow \Delta}{A \lor B, \Gamma \Rightarrow \Delta} \quad \frac{A, \Gamma \Rightarrow \Delta}{\exists x A, \Gamma \Rightarrow \Delta}
\]

State the second rule’s proviso and explain the intuitions behind each rule. [5 marks]

(c) Choose one of these rules and give a rigorous argument for its soundness, using the concept of a valid sequent. [6 marks]

(d) Precisely define the concept of the Most General Unifier (MGU) of two terms, giving examples. [6 marks]