Semantics of Programming Languages

Consider the language below, in which \( r \) ranges over the reals \( \mathbb{R} \) and \( n \) ranges over the subset \( \mathbb{Z} \subset \mathbb{R} \) of integers.

\[
T ::= \text{int} \mid \text{real} \mid \text{bool} \mid T \rightarrow T \mid \{\text{lab} : T, \ldots, \text{lab} : T\}
\]

\[
e ::= n \mid r \mid \text{true} \mid \text{false} \mid \text{fn} x : T \Rightarrow e \mid e e \mid x \mid \{\text{lab} = e, \ldots, \text{lab} = e\} \mid \#\text{lab} \ e
\]

To allow any \text{int} to be used as a \text{real} we can use a subtype relation that satisfies

\[
\text{(num)} \quad \text{int} <: \text{real}
\]

Suppose the typing rules are the standard ones, including the subsumption rule:

\[
\begin{align*}
\text{(sub)} & \quad \frac{\Gamma \vdash e : T \quad T <: T'}{\Gamma \vdash e : T'}
\end{align*}
\]

(a) Give the remainder of the rules required to define the subtype relation \( T <: T' \).

[7 marks]

(b) For each of the following expressions, either give a type derivation (including derivations of any subtype relationships used) or explain why it is not typeable.

\[(i) \quad (\text{fn} \ x : \{p : \text{real}\} \Rightarrow \#p \ x)\{p = 1, q = \text{true}\}
\]

\[(ii) \quad (\text{fn} \ x : \{q : \text{bool}\} \Rightarrow \#p \ x)\{p = 2.7, q = \text{true}\}
\]

\[(iii) \quad (\text{fn} \ x : \{r : \{p : \text{int}, q : \text{bool}\}\} \Rightarrow \#p(\#r \ x))\{r = \{p = 3\}\}
\]

[11 marks]

(c) List all the types \( T \) for which \( \{\} \vdash (\text{fn} \ y : \text{real} \Rightarrow 6) : T \) is derivable.

[2 marks]