Suppose that lists are to be represented in a pure functional manner using a convention where, for instance, a list with three members \( a_1, a_2 \) and \( a_3 \) is modelled by a lambda term

\[
\lambda f.\lambda x.f a_1(f a_2(f a_3 x))
\]

(a) Give the lambda term that corresponds to an empty list. [2 marks]

(b) Explain how the normal list operations can be achieved on lists that are represented in this way. Specifically show how to create pure functional implementations of

(i) a test for an empty list, [2 marks]

(ii) adding a new item to the front of a list, and [4 marks]

(iii) finding the head and tail of a non-empty list. [9 marks]

(c) Show how a map function can be implemented for use with these functional lists, so that

\[
\text{map } f \ [a, b, c] \rightarrow \ [f \ a, f \ b, f \ c]
\]

[3 marks]