

2003 Paper 4 Question 7

Numerical Analysis I

- (a) Explain briefly the *back substitution* algorithm for solving an upper triangular system of linear equations. Why is this important? What is *forward substitution*? [5 marks]
- (b) What is meant by a *symmetric positive definite* matrix? [2 marks]
- (c) Given that $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ is positive definite and

$$A = \begin{pmatrix} 1 & \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ & 1 \end{pmatrix}$$

show how this factorisation may be used to solve the equations

$$A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

[6 marks]

- (d) Now consider the equations

$$\begin{pmatrix} 3 & 4 & 1 \\ 0 & 8 & 2 \\ 3 & 2 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 14 \\ 8 \end{pmatrix}.$$

Pre-multiply each side by $\begin{pmatrix} 1 & 0 & 0 \\ 4 & -1 & -4 \\ 1 & 0 & -1 \end{pmatrix}$ and hence find the solution.

[7 marks]