

2003 Paper 4 Question 6

Continuous Mathematics

- (a) State the definition of the Fourier Transform, $F(\mu)$, of a function $f(x)$ and give the expression for the inverse Fourier Transform of $f(x)$ in terms of $F(\mu)$. [4 marks]

- (b) Consider the function

$$f(x) = \begin{cases} e^{-ax} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

for $a > 0$ and find its Fourier Transform, $F(\mu)$. [4 marks]

- (c) Now consider the function

$$f(x) = e^{-a|x|}$$

for $a > 0$ where $-\infty < x < \infty$ and find its Fourier Transform. [4 marks]

- (d) Show that the Fourier Transform of the function

$$f(x) = \frac{1}{(1+x^2)}$$

is $F(\mu) = \frac{1}{2}e^{-|\mu|}$. [8 marks]