2003 Paper 2 Question 4

Probability

In order to test the integrity of a network of ducting, engineers have developed an inspection device which can be introduced at a node and which then finds its way along a length of ducting to an adjacent node.

In a particular case, eight nodes are sited at the vertices (corners) of a cube and 12 lengths of ducting are arranged along the edges of the cube.

The inspection device is introduced at one node and equiprobably chooses one of the three lengths of ducting leading from that node for its first move. On arrival at the adjacent node the device equiprobably chooses one of the three lengths of ducting leading from that node (including the length it has just inspected). It continues in this fashion until the engineers stop its operation.

Let A_n be the probability of the inspection device returning to the starting node after n moves, and deem $A_0 = 1$.

Let D_n be the probability of the inspection device visiting the node diagonally opposite the starting node after n moves. Clearly, $D_0 = 0$.

(a) Demonstrate that $A_n = 0$ for all odd n and that $D_n = 0$ for all even n. [4 marks]

(b) Determine A_2, A_4 and A_6 expressing all values as fractions. [8 marks]

(c) To what value does A_n tend as (even) n increases indefinitely? [4 marks]

(d) By noting a pattern in the values of A_2, A_4 and A_6 or otherwise, give (without proof) an expression for the value of A_n for arbitrary n. [4 marks]