Complexity Theory

(a) For each $k$, the $k$-clique problem is defined as the following decision problem:

Given a graph $G$, does it contain a clique with at least $k$ vertices?

Show that $k$-clique is in P for each $k$. \hfill [6 marks]

(b) The problem Clique is defined as the following decision problem:

Given a graph $G$ and an integer $k$, does $G$ contain a clique with at least $k$ vertices?

Show that Clique is NP-complete, using the assumption that 3-SAT is NP-complete. \hfill [10 marks]

(c) Explain why, if P=NP then there is a polynomial time algorithm for factorising numbers. \hfill [4 marks]