

## 2003 Paper 12 Question 9

### Numerical Analysis II

- (a) Let  $n_+$  be the number of positive real roots of a polynomial  $p_n(x)$ . Let  $c$  be the number of changes of sign when the coefficients are taken in order. State *Descartes' rule of signs*. [2 marks]
- (b) If  $p_3(x) = x^3 + 13x^2 + 54x + 72$  what does the rule say about the polynomials  $p_3(x)$ ,  $p_3(-x)$ ? [2 marks]
- (c) Using Descartes' rule, a simple search technique and factorisation, find all the real and complex roots of

$$q_5(x) = x^5 + 5x^4 + 32x^3 + 160x^2 + 256x + 1280.$$

[7 marks]

- (d) The Newton–Raphson formula  $\tilde{x} = x - f(x)/f'(x)$  can often be used to find real roots of a polynomial. However, a user of numerical software reports failure to find an accurate root of

$$f(x) = 3x^4 - 28x^3 + 24x^2 + 144x + 432$$

for  $x > 3$  using Newton–Raphson. The user has tried different starting values but gets either a floating-point exception or failure to converge. Using Descartes' rule on  $f(x)$  and  $f'(x)$  investigate the cause of these numerical problems. [9 marks]