Complexity Theory

If \( A \subseteq \Sigma_1^* \) and \( B \subseteq \Sigma_2^* \) are two languages over the alphabets \( \Sigma_1 \) and \( \Sigma_2 \) respectively, we write \( A \leq_P B \) to denote that \( A \) is polynomial-time reducible to \( B \).

(a) Give a precise definition of \( \leq_P \) [2 marks]

(b) Is the relation \( \leq_P \) on languages:

   (i) reflexive?

   (ii) symmetric?

   (iii) transitive?

   Give a proof for your answer in each case. [9 marks]

(c) If \( \Sigma \) is an alphabet, show that if \( P = NP \) then every language \( L \subseteq \Sigma^* \) in \( NP \) is \( NP \)-complete except \( \emptyset \) and \( \Sigma^* \). Why are these two exceptions not \( NP \)-complete? [9 marks]