Data Structures and Algorithms

(a) A million singleton sets each containing a distinct integer are to be successively combined by calls of \texttt{union}(S_1, S_2). The result represents the union of the two disjoint sets represented by \( S_1 \) and \( S_2 \). Interspersed among these calls are several calls of \texttt{inSameSet} where \texttt{inSameSet}(m, n) yields \texttt{true} if and only if \( m \) and \( n \) are integers now in the same set. Describe in detail how you would implement \texttt{union} and \texttt{inSameSet} assuming they will be called about one million and five million times, respectively. Explain why your solution is efficient. [10 marks]

(b) Describe in detail an implementation of Kruskal’s algorithm for finding a minimum cost spanning tree of an undirected graph with positive integer costs on the edges that uses your version of \texttt{union} and \texttt{inSameSet}. [5 marks]

(c) Explain why the spanning tree is unique if all the edge costs are distinct. [5 marks]