Numerical Analysis I

(a) Explain briefly the back substitution algorithm for solving an upper triangular system of linear equations. Why is this important? What is forward substitution? [5 marks]

(b) What is meant by a symmetric positive definite matrix? [2 marks]

(c) Given that \( A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \) is positive definite and

\[
A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}
\]

show how this factorisation may be used to solve the equations

\[
A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.
\]

[6 marks]

(d) Now consider the equations

\[
\begin{pmatrix} 3 & 4 & 1 \\ 0 & 8 & 2 \\ 3 & 2 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 14 \\ 8 \end{pmatrix}.
\]

Pre-multiply each side by \( \begin{pmatrix} 1 & 0 & 0 \\ 4 & -1 & -4 \\ 1 & 0 & -1 \end{pmatrix} \) and hence find the solution. [7 marks]