

## 2002 Paper 9 Question 6

### Types

- (a) Give the typing rules for the polymorphic lambda calculus (PLC). [5 marks]
- (b) Let  $prod(\alpha_1, \alpha_2)$  denote the PLC type  $\forall\alpha((\alpha_1 \rightarrow (\alpha_2 \rightarrow \alpha)) \rightarrow \alpha)$ . Explain how it behaves like the ML product type  $\alpha_1 * \alpha_2$ . To do so, you should give PLC expressions  $pair$ ,  $fst$  and  $snd$  of types

$$\begin{aligned} & \forall\alpha_1(\forall\alpha_2(\alpha_1 \rightarrow (\alpha_2 \rightarrow prod(\alpha_1, \alpha_2)))), \\ & \forall\alpha_1(\forall\alpha_2(prod(\alpha_1, \alpha_2) \rightarrow \alpha_1)), \\ \text{and } & \forall\alpha_1(\forall\alpha_2(prod(\alpha_1, \alpha_2) \rightarrow \alpha_2)) \end{aligned}$$

respectively, corresponding to the ML polymorphic pairing and projection functions

$$\begin{aligned} & \text{fn } x1 \Rightarrow \text{fn } x2 \Rightarrow (x1, x2) , \\ & \text{fn } (x1, x2) \Rightarrow x1 , \\ \text{and } & \text{fn } (x1, x2) \Rightarrow x2 . \end{aligned}$$

Give proofs for the typing of  $pair$  and  $fst$ , and explain the beta-conversion properties of  $fst \tau_1 \tau_2 (pair \tau_1 \tau_2 M_1 M_2)$ , for any PLC types  $\tau_1, \tau_2$  and terms  $M_1, M_2$ . [10 marks]

- (c) Is it always the case that a PLC term  $M$  of type  $prod(\tau_1, \tau_2)$  is beta-convertible to  $pair \tau_1 \tau_2 (fst \tau_1 \tau_2 M) (snd \tau_1 \tau_2 M)$ ? Justify your answer. [Hint: consider terms with free variables.] [5 marks]