Information Theory and Coding

(a) (i) A Hamming Code allows reliable transmission of data over a noisy channel with guaranteed error correction as long as no more than one bit in any block of 7 is corrupted. What is the maximum possible rate of information transmission, in units of (data bits reliably received) per (number of bits transmitted), when using such an error correcting code? [2 marks]

(ii) In such a code, what type of Boolean operator on the data bits is used to build the syndromes? Is this operator applied before transmission, or upon reception? [2 marks]

(b) (i) For each of the four classes of signals in the following table,

<table>
<thead>
<tr>
<th>Class</th>
<th>Signal Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>continuous, aperiodic</td>
</tr>
<tr>
<td>2.</td>
<td>continuous, periodic</td>
</tr>
<tr>
<td>3.</td>
<td>discrete, aperiodic</td>
</tr>
<tr>
<td>4.</td>
<td>discrete, periodic</td>
</tr>
</tbody>
</table>

identify its characteristic spectrum from the following table:

<table>
<thead>
<tr>
<th>Class</th>
<th>Spectral Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>continuous, aperiodic</td>
</tr>
<tr>
<td>B.</td>
<td>continuous, periodic</td>
</tr>
<tr>
<td>C.</td>
<td>discrete, aperiodic</td>
</tr>
<tr>
<td>D.</td>
<td>discrete, periodic</td>
</tr>
</tbody>
</table>

(Give your answer just in the form 1-A, 2-B, etc. Note that you have 24 different possibilities.) [8 marks]

(ii) For each case, name one example of such a function and its Fourier transform. [4 marks]

(c) Give two reasons why Logan’s Theorem about the richness of zero-crossings for encoding and recovering all the information in a one-octave signal may not be applicable to images as it is for one-dimensional signals. [4 marks]