

2002 Paper 8 Question 11

Numerical Analysis II

- (a) In the Chebyshev characterisation theorem, the best L_∞ approximation to $f(x)$ over a closed finite interval by a polynomial $p_{n-1}(x)$ of degree $n - 1$ has the property that the maximum error $|e(x)|$ is attained at M distinct points ξ_j such that $e(\xi_j) = -e(\xi_{j-1})$. What is M ? [2 marks]
- (b) Let $x = m \times 2^k$ represent a normalised number in a floating-point implementation. When computing \sqrt{x} show how the domain of the problem can be reduced to $x \in [1, 4)$. Find the coefficients a, b which minimise $\|e(x)\|_\infty$ over $[1, 4]$ where $e(x) = ax + b - \sqrt{x}$. [8 marks]
- (c) Taking full account of symmetry, describe the form of the best polynomial approximation $p(x)$ to x^4 over $[-1, 1]$ by a polynomial of lower degree. Sketch x^4 and $p(x)$, showing the extreme values of $|e(x)|$ where $e(x) = x^4 - p(x)$. Hence compute the coefficients of $p(x)$. [10 marks]