2002 Paper 7 Question 12

Information Theory and Coding

- (a) State and explain (without proving) two different theorems about signal encoding that both illustrate the following principle: strict bandlimiting (either lowpass or bandpass) of a continuous signal reduces the information that it contains from potentially infinite to a finite discrete set of data, and allows exact reconstruction of the signal from just a sparse set of sample values. For both of your examples, explain what the sample data are, and why bandlimiting a signal has such a dramatic effect on the amount of information required to represent it completely. [10 marks]
- (b) A variable length, uniquely decodable code which has the prefix property, and whose N binary code word lengths are

$$n_1 \leqslant n_2 \leqslant n_3 \cdots \leqslant n_N$$

must satisfy what condition on these code word lengths?

(State both the condition on the code word lengths, and the name for this condition, but do not attempt to prove it.) [4 marks]

(c) For a discrete data sequence consisting of the N uniformly-spaced samples

$$\{g_n\} = \{g_0, g_1, \ldots, g_{N-1}\}$$

define both the Discrete Fourier Transform $\{G_k\}$ of this sequence, and its Inverse Transform, which recovers $\{g_n\}$ from $\{G_k\}$. [6 marks]