

2002 Paper 5 Question 11

Logic and Proof

- (a) For each of the following formulae, state (with justification) whether it is satisfiable, valid or neither:

$$((Q \rightarrow R) \rightarrow Q) \wedge \neg Q \quad [2 \text{ marks}]$$

$$((P \leftrightarrow Q) \leftrightarrow P) \leftrightarrow Q \quad [2 \text{ marks}]$$

$$\exists xy [P(x, y) \rightarrow \forall xy P(x, y)] \quad [3 \text{ marks}]$$

$$[\forall x (P(x) \rightarrow Q(x)) \wedge \exists x P(x)] \rightarrow \forall x Q(x) \quad [3 \text{ marks}]$$

- (b) Briefly outline the semantics of first-order logic, taking as an example the formula $\forall xy f(x, y) = f(y, x)$. [6 marks]

- (c) Exhibit a model that satisfies both of the following formulae (a is a constant):

$$\forall x g(x) \neq a$$

$$\forall xy [g(x) = g(y) \rightarrow x = y]$$

[4 marks]