Continuous Mathematics

Suppose that $F(\mu)$ is the Fourier transform of the function $f(x)$.

(a) State the integral expression for $F(\mu)$ in terms of $f(x)$ and the inverse transform for $f(x)$ in terms of $F(\mu)$. [2 marks]

(b) Determine the **shift rule** for the Fourier transform of $f(x - \alpha)$ where $\alpha$ is a constant. [3 marks]

(c) Determine the **scale rule** for the Fourier transform of $f(\alpha x)$ where $\alpha$ is a non-zero constant. [3 marks]

(d) Given that the standard normal density function $\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ has Fourier transform $F(\mu)$ use the scale and shift rules to determine the Fourier transform of the normal density $\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\gamma)^2/(2\sigma^2)}$. Here $\gamma$ is a real number and $\sigma$ is a positive real number. [6 marks]

(e) Show that

$$G(\mu) = A \int_{-\infty}^{\infty} g(x)e^{-ia\mu x} \, dx$$

implies that

$$g(x) = \frac{|a|}{2\pi A} \int_{-\infty}^{\infty} G(\mu)e^{ia\mu x} \, d\mu$$

for all non-zero constants $a$ and $A$. You may assume that the result holds in the special case when $a = 1$ and $A^{-1} = 2\pi$. [6 marks]