2002 Paper 4 Question 5

Continuous Mathematics

Suppose that $F(\mu)$ is the Fourier transform of the function $f(x)$.

(a) State the integral expression for $F(\mu)$ in terms of $f(x)$ and the inverse transform for $f(x)$ in terms of $F(\mu)$. [2 marks]

(b) Determine the *shift rule* for the Fourier transform of $f(x-\alpha)$ where α is a constant. [3 marks]

(c) Determine the *scale rule* for the Fourier transform of $f(\alpha x)$ where α is a non-zero constant. [3 marks]

(d) Given that the standard normal density function $\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ has Fourier transform $F(\mu)$ use the scale and shift rules to determine the Fourier transform of the normal density $\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\gamma)^2/(2\sigma^2)}$. Here γ is a real number and σ is a positive real number. [6 marks]

(e) Show that

$$G(\mu) = A\int_{-\infty}^{\infty} g(x)e^{-ia\mu x}\,dx$$

implies that

$$g(x) = \frac{|a|}{2\pi A}\int_{-\infty}^{\infty} G(\mu)e^{ia\mu x}\,d\mu$$

for all non-zero constants a and A. You may assume that the result holds in the special case when $a = 1$ and $A^{-1} = 2\pi$. [6 marks]