Foundations of Computer Science

This question concerns the following ML declaration of a tree datatype:

```ml
datatype 'a fan = Wave of 'a * ('a fan) list;
```

(a) Declare the function `flip`, which maps a tree to a mirror image of itself, as illustrated:

```
1
|   |
2   3
|--|--|
5   6
```

```
1
|   |
4   3
|--|--|
2   1
```

(b) Declare the curried function `paint f`, which copies a tree while applying the function `f` to each of its labels.

(c) Declare the function `same_shape`, which compares two trees and returns `true` if they are equal except for the values of their labels and otherwise returns `false`.

(d) State the types of functions `flip`, `paint` and `same_shape`.

(e) The function `paper` is declared in terms of the familiar functional `foldr`:

```ml
fun foldr f ([], e) = e
  | foldr f (x::xs, e) = f(x, foldr f (xs,e));

fun paper (Wave(x,fs), q) = foldr paper (fs, q+1);
```

Describe the computation that results when `paper` is applied to a tree.