# 2002 Paper 13 Question 9

## Numerical Analysis II

Consider the alternative formulae

$$y_{n+1} = y_n + hf(x_n, y_n) + O(h^2) \tag{1}$$

$$y_{n+1} = y_{n-1} + 2hf(x_n, y_n) + O(h^3) \tag{2}$$

applied to the ODE

$$y' = -5y, \quad y(0) = 1$$

using $h = 0.1$ in each case.

($a$) Define the terms *local error* and *order* for an ODE formula. What is the *order* of each of the methods (1) and (2)? [2 marks]

($b$) Giving answers to 2 significant decimal digits of accuracy, compute the solution of the ODE for $x_n = 0, 0.1, 0.2, \ldots$ 1.0 for each method. Tabulate your answers. The exact solutions to 2 significant digits are:

1.0, 0.61, 0.37, 0.22, 0.14, 0.082, 0.050, 0.030, 0.018, 0.011, 0.0067

Assume the exact value of $y(0.1)$ for method (2). [7 marks]

($c$) Which method is more accurate initially and why? Explain the behaviour of each method as $x$ increases. [3 marks]

($d$) Solve the ODE. Find a general term for $y_n$ in method (1) and show that the absolute error in (1) will be small when $n$ is large. Without performing any further calculations, how do you expect the absolute error in method (2) to behave when $n$ is large? [5 marks]

($e$) Discuss briefly the suitability of formulae (1) and (2) as predictors for predictor–corrector methods in respect of *order* and *stability.* [3 marks]