Complexity Theory

(a) Give a precise definition of what is meant by the statement that a decision problem $A$ is \textit{polynomial-time reducible} to a decision problem $B$. [2 marks]

(b) Consider the following three decision problems on graphs.

- \textbf{Connect} — the collection of graphs $G$ such that there is a path between any two vertices of $G$.
- \textbf{Hamilton} — the collection of graphs that contain a Hamiltonian cycle.
- \textbf{non-3-colour} — the collection of graphs that are not 3-colourable.

For each of the following statements, state whether it is true, false or an unresolved open question. Give a brief justification for your answer.

(i) \textbf{Connect} is decidable by a polynomial time algorithm.

(ii) \textbf{Hamilton} is decidable by a polynomial time algorithm.

(iii) \textbf{non-3-colour} is decidable by a polynomial time algorithm.

(iv) \textbf{Connect} is polynomial-time reducible to \textbf{Hamilton}.

(v) \textbf{Hamilton} is polynomial-time reducible to \textbf{non-3-colour}.

(vi) \textbf{non-3-colour} is polynomial-time reducible to \textbf{Connect}.

[3 marks each]