

2001 Paper 6 Question 11

Logic and Proof

- (a) In the context of clause-based proof methods, define the notion of *pure literal* and describe what should be done if the set of clauses contains pure literals. [3 marks]
- (b) Use the Davis–Putnam method to discover whether the following set of clauses is satisfiable. If they are satisfiable, show a satisfying interpretation.

$$\{P, R\} \quad \{\neg P, \neg R\} \quad \{P, \neg Q\} \quad \{\neg Q, R\} \quad \{\neg P, Q, R\}$$

[6 marks]

- (c) The three-fingered inhabitants of the planet Triterra build base-3 computers. A Triterran named Randal Tryant has found a way of verifying base-3 combinational logic. His Ordered Ternary Decision Diagrams (OTDDs) are the same as a technology used on planet Earth except that all variables and expressions range over the values 0, 1 and 2 instead of just 0 and 1.
- (i) Describe how a full ternary decision tree can be reduced to an OTDD without regard for efficiency. [2 marks]
- (ii) Sketch an efficient algorithm to convert a ternary expression directly to an OTDD without constructing the full decision tree. For a typical ternary connective use modulo-3 multiplication, written as \otimes . [6 marks]
- (iii) Demonstrate your algorithm by applying it to the ternary expression $((i \otimes i) \otimes j) \otimes 2$. [3 marks]